Deriving SAM Multiplier Models Using IMPLAN

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Abstract

Multiplier models based upon input-output and social accounts (SAMs) derived using IMPLAN are described. Modeling induced household effects using IMPLAN's traditional Type III formulation is compared with a classic Type II model using SAM accounts. A shortcoming in the SAM accounts is noted that restricts the evaluation of income distribution effects..

Input-Output and Social Accounts Constructed with IMPLAN

Regional input-output and social accounts are constructed with the IMPLAN system utilizing various data, estimation methods and algorithms. These accounts conform to specific conventions that determine the types of multiplier models that can be derived from them. This section describes the various accounts that can be developed using IMPLAN.

Figure 1 illustrates the standard accounting system for the input-output accounts of IMPLAN, utilizing the detailed "industry and commodity" format (the Appendix describes the specific contents of the sub-matrices for these accounts). The input-output accounts, while very detailed in the treatment of the production and consumption components of an economy, do lack completeness in the area of nominal transactions occurring between institutions (e.g., transfers, taxes, savings). This shortcoming is addressed by expanding the accounting system as is illustrated in Figure 2, the social accounting format of IMPLAN. The input-

Figure 1. Input-Output Accounting Framework, Industry and Commodity Format

	Industries	Commodities	Institutions	Trade	Total
Industries		Domestic Commodity Make		Commodity Exports	Industry Output
Commodities	Domestic Commodity Use		Domestic Commodity Consumption		Domestic Commodity Output
Factors	Value Added				Value Added
Institutions		Domestic Commodity Sales		Commodity Exports	Institutional Receipts from Commodity Sales
Trade	Commodity Imports		Commodity Imports	Transshipments	Commodity Imports
Total	Industry Outlay	Domestic Commodity Output	Institutional Outlay for Commodities	Commodity Exports	

output accounts are augmented with two principal sub-matrices (shaded in the Figure), one for factor disbursements and another for inter-institutional transfers. This has the effect, along with concordant accounting of factor and institutional trade, of "completing the income cycle." Given these additions, it is possible to trace income flows from income generation, through factor receipts and their disbursement to receiving institutions, observe inter-institutional transactions to account for all sources of institutional

Figure 2. Social Accounting Framework, Industry and Commodity Format

	Industries	Commodities	Factors	Institutions	Trade	Total
Industries		Domestic Commodity Make			Commodity Exports	Industry Output
Commodities	Domestic Commodity Use			Domestic Commodity Consumption		Domestic Commodity Output
Factors	Value Added					Value Added
Institutions		Domestic Commodity Sales	Factor Disbursements	Inter-Institutional Transfers	Commodity and Factor Services Exports	Institutional Receipts
Trade	Commodity Imports		Factor Services Imports	Commodity Imports and Remittances	Transshipments	Commodity, Factor Services Imports, Remittances
Total	Industry Outlay	Domestic Commodity Output	Factor Outlay	Institutional Outlay	Commodity, Factor Services Exports	

income, and relate that income to consumption by institutions. In sum, the social accounting format captures all current account transactions for a regional economy unlike input-output accounts which focus primarily on the production and consumption components.

Input-output accounts can be derived from the accounts shown in Figure 2 by applying assumptions about inter-industry transaction behavior. From the accounts shown in Figure 2, define two matrices of industry (and institution) commodity sales:

(1) M the "make" of commodities for domestic consumption, and
(2) J the "make" of commodities for export,
where both matrices have dimension $(m + n) \times c$ and
m is the number of industries,
n is the of institutions, and
c is the number of commodities.

Each of the matrices M and J can be thought of as partitioned matrices, such as

(4) $$M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$$ and

(5) $$J = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}$$

where subscript *1* relates to industries (*i.e.*, sales of commodities by industries) and subscript *2* refers to institutions (*i.e.*, sales of commodities by institutions). The accounting balance for total commodity output q is:

(6) $$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = i \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} + i \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}$$

where i is a summation vector.

A market-shares matrix D can be derived for domestic consumption by post-multiplying the make matrix by a diagonal matrix of the reciprocals of total commodity output $\hat{q}^{-1}$:

(7) $$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \hat{q}^{-1}$$

Similarly, a market-shares matrix E for exports can be derived:

(8) $$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \hat{q}^{-1}$$

Applying the market-share hypothesis to derive inter-industry transactions by pre-multiplying the domestic use matrix U by the domestic market-shares matrix D by the produces a domestic inter-industry transactions matrix X :

(9) $$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} U$$

Again, the subscript *1* refers to industries and subscript *2* refers to institutions.

Figure 3 illustrates the social accounting system after the input-output components have been combined to describe inter-industry transactions. From equation (9) above, sub-matrix X_1 represents the domestic inter-industry transactions matrix and sub-matrix X_2 the institutional sales to industries transactions matrix. Again using the market-share hypothesis to estimate institutional consumption transactions, a domestic institutional transactions matrix C can be derived by pre-multiplying the domestic commodity consumption matrix (final demands) F by the domestic market-shares matrix D :

(10) $$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} F$$

From equation (10) above, sub-matrix C_1 refers to the domestic institutional consumption transactions matrix (final demands) and sub-matrix C_2 refers to institutional sales of commodities to institutions (transactions that must be inserted into the inter-institutional transfers matrix).

Figure 3. Social Accounting Framework, Inter-industry Format

	Industries	**Factors**	**Institutions**	**Trade**	**Total**
Industries	Domestic Inter-industry Transactions X_1		Domestic Consumption C_1	Exports of Industry Output G_1	Industry Output
Factors	Value Added				Value Added
Institutions	Institutional Sales to Industries X_2	Factor Disbursements	Inter-Institutional Commodity Sales C_2 and Transfers	Institutional Exports G_2 and Factor Services	Institutional Receipts
Trade	Commodity Imports	Factor Services Imports	Commodity Imports and Remittances	Transshipments	Commodity, Factor Services Imports, Remittances
Total	Industry Outlay	Factor Outlay	Institutional Outlay	Industry, Factor Services Exports	

Finally, again using the market-share hypothesis, an export transactions matrix G containing industrial (and institutional) exports can be derived by pre-multiplying the export matrix J by the export market-shares matrix E :

(11) $\begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} J$

From equation (11) above, sub-matrix G_1 refers to the industrial export transactions matrix and sub-matrix G_2 refers to institutional exports of commodities.

Modeling the Induced Effect with IMPLAN I-O Accounts

While the inter-industry part of IMPLAN models is quite traditional and widely accepted, the component of the multipliers dealing with induced re-spending of household income is less so. This section describes the formulation of the Type III multiplier model in the context of IMPLAN's traditional accounting structure comparing it to a Type II formulation.

Following standard derivation, the I-O balance equation based on the accounting framework in Figure 3 is:

(12) $x = X + y$ where

m = the number of domestic industries,

x is a $(m+1)$ x *1* vector of total output, including household output,

X is the $(m+1)$ x $(m+1)$ inter-industry transactions matrix (as in (9) above), and also including a household row and column, and

y is final demand including exports but less household demands, reduced to a vector.

Assuming inputs are proportional to total outlays, a direct input requirements matrix A can be derived:

(13) $A = X\hat{x}^{-1}$ such that

(14) $X = Ax$

By substitution, the standard behavioral model for inter-industrial multiplier analysis can be written as:

(15) $x = Ax + y$ or, solving for x as a function of y,

(16) $x = (I - A)^{-1} y$

Figure 4. I-O Model Framework, Inter-industry Format

	Industries	**Households**	**Other Institutions and Trade**	**Total**
Industries	Domestic Direct Coefficients A_{11}	Domestic Household Coefficients A_{12}	Final Demand and Exports y_1	Industry Output x_1
Factors	Value Added Coefficients A_{21}	A_{22}	y_2	Value Added x_2

Focusing in on the household component, the system of equation (15) can be partitioned into sub-matrices, as shown in Figure 4, as follows:

(17) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

where the subscript *1* refers to industries and the subscript *2* refers to households. Equation (17) can be rewritten as a system of equations:

(18) $x_1 = A_{11} x_1 + A_{12} x_2 + y_1$

(19) $x_2 = A_{21}x_1 + A_{22}x_2 + y_2$

Rearranging equation (18) in terms of x_1 yields:

(20) $x_1 = (I - A_{11})^{-1} A_{12} x_2 + (I - A_{11})^{-1} y_1$

Similarly, rearranging equation (19) in terms of x_2 yields:

(21) $x_2 = (I - A_{22})^{-1} A_{21} x_1 + (I - A_{22})^{-1} y_2$

Given that y_1 and y_2 are known (*i.e.*, exogenous demands are known), the system of equations (20) and (21) can be solved algebraically by substituting equation (21) into equation (20) or, as is done algorithmically with IMPLAN, the system can be solved by approximation.

The difficulty with the formulation above lies in the definition of the sub-matrices of A relating to households, specifically sub-matrices A_{12}, A_{21} and A_{22}. By definition, the formulation above presumes that sub-matrix A_{12} represents normalized household consumption, sub-matrix A_{22} is an interaction term, and sub-matrix A_{21} represents normalized household income coefficients. Unfortunately, sub-matrix A_{21} cannot be derived from the "traditional" IMPLAN input-output accounts of the type shown in Figure 1. Close inspection will show that the coefficients of sub-matrix A_{21} are derived from the value added account which represent income payments by industries classified according to factor groups (such as employee compensation and property income). In contrast, sub-matrix A_{12}, household consumption is classified according to the institution (households) making the purchase. The accounts do not contain enough information to separate household income from gross factor income. Further, household "place of residence" income cannot be distinguished from the gross "place of work" factor payments. Thus the disparate classifications of income as "place of work" factor receipts on one hand and consumption as household "place of residence" spending precludes the direct construction of the coefficients for sub-matrix A_{21} consistent with the other household sub-matrices of A.

These shortcomings for modeling the induced effects of household spending in the traditional IMPLAN accounting system were addressed by deriving a Type III multiplier. The accounting system was modified as shown in Figure 5, changing the definition of sub-matrices A_{12}, A_{21} and A_{22}. In this case, sub-matrix A_{12} is redefined as normalized *per capita* household consumption, sub-matrix A_{22} remains an interaction term, and sub-matrix A_{21} is redefined as normalized coefficients representing the resident population receiving income (as a function of employment). Redefining sub-matrix A_{21} in this way resolves two shortcomings noted above on the presumption that household income is closely aligned with employment and the difference between "place of work" and "place of residence" income can be explained by focusing on the resident population. Using these redefinitions of A, the model described by equations (20) and (21) above can be solved as a system with the induced component formulated as a Type III multiplier.

Figure 5. Modified I-O Model Framework, Inter-industry Format

	Industries	Households	Other Institutions and Trade	Total
Industries	Domestic Direct Coefficients A_{11}	Domestic Household Consumption Coefficients (per capita) A_{12}	Final Demand and Exports y_1	Industry Output x_1
Households	Population receiving income (function of employment) A_{21}	A_{22}	y_2	Population x_2

Modeling the Induced Effect with IMPLAN SAM Accounts

The expansion of the IMPLAN accounts to a complete social accounting system overcomes the limitations of the I-O accounts related to modeling induced effects. This section describes the formulation of the Type II multiplier model in the context of IMPLAN's SAM accounting structure.

Following the accounting framework shown in Figure 6, the SAM balance equation based on the accounting framework in Figure 3 is:

(22) $x = X + y$ where

m = the number of domestic industries,
n = the number of factors,
x is a $(m + n + 1)$ x 1 vector of total industry and factor output, plus household output,
X is a $(m + n + 1)$ x $(m + n + 1)$ transactions matrix including sub-matrices of inter-industrial transactions, value added, factor disbursements and inter-institutional transfers and also including households, and
y is final demand including exports but less household demands, reduced to a vector.

Assuming inputs are proportional to total outlays, a direct input requirements matrix A can be derived:

(23) $A = X\hat{x}^{-1}$ such that

(24) $X = Ax$

Figure 6. SAM Model, Type II Formulation

	Industries	Factors	Households	Other Institutions and Trade	Total
Industries	Direct Requirements Coefficients A_{11}		Household Consumption Coefficients A_{13}	Institutional Consumption and Exports y_1	Industry Output x_1
Factors	Value Added A_{21}			y_2	Value Added x_2
Households	Direct Requirements from Households A_{31}	Factor Disbursement Coefficients A_{32}	Inter-Household Transfers A_{33}	Inter-Institutional Transfers, Institutional, Factor Services Exports y_3	Institutional Receipts x_3

By substitution, the standard behavioral model for general multiplier analysis can be written as:

(25) $x = Ax + y$ or, solving for x as a function of y,

(26) $x = (I - A)^{-1} y$

Focusing in on the income flows from the production to factors and ultimately disbursement to households, the system can be partitioned into sub-matrices as follows:

$$(27) \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} A_{11} & & A_{13} \\ A_{21} & & \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

where the subscript *1* refers to industries, the subscript *2* refers to factors, and the subscript *3* refers to households. Equation (27) can be rewritten as a system of equations:

(28) $x_1 = A_{11} x_1 + A_{13} x_3 + y_1$

(29) $x_2 = A_{21} x_1 + y_2$

(30) $x_3 = A_{31} x_1 + A_{32} x_2 + A_{33} x_3 + y_3$

Rearranging equation (28) in terms of x_1 yields:

(31) $x_1 = (I - A_{11})^{-1} A_{13} x_3 + (I - A_{11})^{-1} y_1$

Equation (29) already gives the solution for x_2 :

(32) $x_2 = A_{21} x_1 + y_2$

Rearranging equation (30) in terms of x_3 yields:

(33) $x_3 = (I - A_{33})^{-1} A_{31} x_1 + (I - A_{33})^{-1} A_{32} x_2 + (I - A_{33})^{-1} y_3$

Given that y_1 , y_2 and y_3 are known (*i.e.*, exogenous changes in demand or income are known), the system of equations (31), (32) and (33) can be solved algebraically by substituting equations (31) and (32) into equation (33) or, as is done algorithmically with IMPLAN, the system can be solved by approximation.

This derivation, based on the expanded accounting system, explicitly solves for the Type II formulation of induced effects. The two shortcomings of the I-O accounting structure, inconsistent classification between household income and consumption and the lack of correction from "place of work" to "place of residence" income, are both eliminated in the SAM formulation. Sub-matrix A_{32} is the key component, describing how gross "place of work" factor receipts (sub-matrix A_{21}) are allocated to domestic institutions (net of imported factor services) as "place of residence" income for factor services.

Two additional comments can be made about this formulation. The first is, perhaps, obvious. The Type II formulation described above limits the induced component of the multiplier model to household spending only. Arguments can be easily made, however, that other institutions should be included in the induced component, such as parts or all of government operations. Expanding the dimensions of the A matrix to accommodate these institutions as desired would result in the derivation of any of several Type *n* multipliers depending on which institutions are included. A second comment concerns an underlying simplicity in the SAM. Consider the relationship between factor receipts and their disbursement to institutions in the formulation above. Total factor receipts x_2 are determined as in equation (28) and total factor receipts are distributed among institutions by the term $A_{32} x_2$ in equation (29). The point is that the interaction term in the equations has to do with *total* factor receipts (the sub-vector x_2) rather than a *matrix* of factor receipts showing variations in factor earnings across industries. Thus the *size* distribution of factor incomes among institutions must be fixed (given by the sub-matrix A_{32}) regardless of the industry in which the factor income may originate. In other words, an impact model derived from these accounts will be able to capture variations in the *factor* distribution of income (different industries may use individual factors in

different proportions), but not the *size* distribution of income among institutions. In this sense, I consider the SAM somewhat "brain-dead", lacking an appealing attribute for examining changes in the size distribution of income. Only with the addition of transition sub-matrices to link sectoral factor incomes to institutions could the deficiency be overcome.

Appendix A: Sub-matrix Definitions for the IMPLAN Accounts

Figure A1. Input-Output Accounting Framework, Industry and Commodity Format

	Industries	Commodities	Institutions	Trade	Total
Industries		Domestic Commodity Make (a)		Commodity Exports (b)	Industry Output
Commodities	Domestic Commodity Use (c)		Domestic Commodity Consumption (d)		Domestic Commodity Output
Factors	Value Added (e)				Value Added
Institutions		Domestic Commodity Sales (f)		Commodity Exports (i)	Institutional Receipts from Commodity Sales
Trade	Commodity Imports (j)		Commodity Imports (l)	Transshipments (m)	Commodity Imports
Total	Industry Outlay	Domestic Commodity Output	Institutional Outlay for Commodities	Commodity Exports	

Sub-matrix definitions for the input-output accounting framework, industry and commodity format:

Sub-matrix	Label	Description
(a)	Domestic Commodity Make	Production of commodities by domestic industries for domestic consumption
(b)	Commodity Exports	Production of commodities by domestic industries for export
(c)	Domestic Commodity Use	Consumption of domestically produced commodities by domestic industries
(d)	Domestic Commodity Consumption	Consumption of domestically produced commodities by domestic institutions
(e)	Value Added	Factor payments by domestic industries
(f)	Domestic Commodity Sales	Commodity sales by domestic institutions
(i)	Commodity Exports	Commodity exports by domestic institutions
(j)	Commodity Imports	Commodity imports by domestic industries
(l)	Commodity Imports	Commodity imports by domestic institutions
(m)	Transshipments	Commodity imports for export

Figure A2. Social Accounting Framework, Industry and Commodity Format

	Industries	Commodities	Factors	Institutions	Trade	Total
Industries		Domestic Commodity Make (a)			Commodity Exports (b)	Industry Output
Commodities	Domestic Commodity Use (c)			Domestic Commodity Consumption (d)		Domestic Commodity Output
Factors	Value Added (e)					Value Added
Institutions		Domestic Commodity Sales (f)	Factor Disbursements (g)	Inter-Institutional Transfers (h)	Commodity and Factor Services Exports (i)	Institutional Receipts
Trade	Commodity Imports (j)		Factor Services Imports (k)	Commodity Imports and Remittances (l)	Transshipments (m)	Commodity, Factor Services Imports, Remittances
Total	Industry Outlay	Domestic Commodity Output	Factor Outlay	Institutional Outlay	Commodity, Factor Services Exports	

Sub-matrix definitions for the social accounting framework, industry and commodity format:

Sub-matrix	Label	Description
(a)	Domestic Commodity Make	Production of commodities by domestic industries for domestic consumption
(b)	Commodity Exports	Production of commodities by domestic industries for export
(c)	Domestic Commodity Use	Consumption of domestically produced commodities by domestic industries
(d)	Domestic Commodity Consumption	Consumption of domestically produced commodities by domestic institutions
(e)	Value Added	Factor payments by domestic industries
(f)	Domestic Commodity Sales	Commodity sales by institutions
(g)	Factor Disbursements	Factor disbursements to domestic institutions
(h)	Inter-institutional Transfers	Transactions between domestic institutions
(i)	Commodity and Factor Services Exports	Commodity exports by domestic institutions and factor service exports by institutions
(j)	Commodity Imports	Commodity imports by domestic industries
(k)	Factor Services Imports	Factor services imports by domestic industries
(l)	Commodity Imports and Remittances	Commodity imports by domestic institutions and remittances by domestic institutions to the rest of the world
(m)	Transshipments	Commodity imports for export

Figure A3. Social Accounting Framework, Interindustry Format

	Industries	**Factors**	**Institutions**	**Trade**	**Total**
Industries	Domestic Inter-industry Transactions (o)		Domestic Consumption (p)	Exports of Industry Output (q)	Industry Output
Factors	Value Added (e)				Value Added
Institutions	Institutional Sales to Industries (r)	Factor Disbursements (g)	Inter-Institutional Transfers (h)	Institutional and Factor Services Exports (i)	Institutional Receipts
Trade	Commodity Imports (j)	Factor Services Imports (k)	Commodity Imports and Remittances (l)	Transshipments (m)	Commodity, Factor Services Imports, Remittances
Total	Industry Outlay	Factor Outlay	Institutional Outlay	Industry, Factor Services Exports	

Sub-matrix definitions for the social accounting framework, inter-industry format:

Sub-matrix	Label	Description
(e)	Value Added	Factor payments by domestic industries
(g)	Factor Disbursements	Factor disbursements to domestic institutions
(h)	Inter-institutional Transfers	Transactions between domestic institutions
(i)	Commodity and Factor Services Exports	Commodity exports by domestic institutions and factor service exports by institutions
(j)	Commodity Imports	Commodity imports by domestic industries
(k)	Factor Services Imports	Factor services imports by domestic industries
(l)	Commodity Imports and Remittances	Commodity imports by domestic institutions and remittances by domestic institutions to the rest of the world
(m)	Transshipments	Commodity imports for export
(o)	Domestic Inter-industry Transactions	Transactions between domestic industries
(p)	Domestic Consumption	Consumption by domestic institutions from domestic industries
(q)	Exports of Industry Output	Exports by domestic industries
(r)	Institutional Sales to Industries	Commodity sales by domestic institutions to domestic industries