White Paper

Economic Impact, Economic Contribution, and Export Base

Huntersville, NC

IMPLAN

White Paper

Economic Impact, Economic Contribution, and Export Base

Huntersville, NC

IMPLAN

Table of Contents

01 Introduction

02 Economic Impact Analysis

03 Contribution Analysis – Extraction Method

05 Contribution Analysis – Gross-Base Method

08 Technical Section: Internal Consistency of IMPLAN's Contribution Analysis Methods

12 Figures

16 References

Introduction

The purpose of this paper is to clarify the differences between three commonly confused terms and methods in the more general realm of input-output (I-O) analysis: economic impact analysis, economic contribution analysis, and gross-base analysis. While each of the three types of analyses has a distinct purpose and method, they all rely on data from an I-O model, which itself is a bit of a misnomer. When first popularized by Wassily Leontief, I-O tables focused on the purchasing (input) and production (output) behavior of industries only. Later, the purchasing, selling, saving, and investing behavior of institutions such as governments and households was also added to the I-O tables, forming what is now known as a Social Accounting Matrix (SAM) and spurring a corresponding name change from I-O models to SAM models.

In the discussion that follows, industry output is used as the measure of interest, though the same principles apply to employment and each of the individual components of output. We have included a technical section that gives a detailed numerical demonstration of IMPLAN's recommended contribution analysis methods and their relationship to the established literature.

Economic Impact Analysis

In a common final demand change (i.e., “impact”) analysis, the analyst is modeling a new firm or a change in the level of output of an existing firm. In such cases, the Direct Effect is the new firm’s total output or the existing firm’s change in output. In such cases, it is likely (and logical) that the industry in which that firm belongs will experience total impacts that exceed the direct impacts – that is, the industry will experience indirect and induced effects that stem from the direct effects.

Consider the example of wood window and door manufacturing in Washington County, MN. This industry already exists in the county, with Anderson Windows being one of the existing firms in this industry in Washington County. Now suppose a new wood windows and doors manufacturer, Wonder Windows, is planning to locate in Washington County with expected output of $5 million dollars. This new plant would create new demand for some local businesses (the purchase of dimension lumber and preserved wood products, for example), which may spur some of these local businesses to undertake building improvements or expansions, thereby necessitating new windows, thereby creating indirect demand for the wood windows and doors sector. Additionally, most firms make purchases from other firms in their own industry (for example, consulting services, equipment rental), which generates additional indirect demand for that same industry. In addition to these indirect effects, the new Wonder Windows plant would also attract some new workers to the region, spurring home improvements to some existing homes and apartments, thereby generating induced demand for the wood windows and doors industry. Such “feedback” effects to the industry are part of that industry’s overall impact in the region.

Contribution Analysis – Extraction Method

However, there are occasions when an analyst would like to see the indirect and induced effects that the current level of output of an existing industry as a whole has on other industries in that region. In this case, the goal is to generate a total output effect on the primary industry of interest that is equal to the current level of production of that industry, while showing the indirect and induced effects that this current level of output has on other industries in the region. In other words, the only "effects" that the industry of interest should experience are the direct effects (e.g., current output), while other industries in the region experience indirect and induced effects associated with (i.e., in support of) the direct effects in the industry under study. In such a study, it does not make sense to allow feedback effects on the primary industry of interest, since an existing Industry cannot experience a total output "effect" that exceeds its current level of output. In other words, the question of contribution analysis definitionally assumes that the effect on the sector whose contribution is being measured is the entire value of its own output.[1]

As another example, consider the shutting down of an industry.   If the current level of industry output were to be modeled as a negative direct effect using the traditional impact analysis approach, the model would show a total loss to this industry that is greater than its current level of output – but it is not possible to lose more output (or employment) than currently exists!

Therefore, special modeling techniques are required in these cases to ensure that the results accurately reflect the current level of output of the industry of interest plus the indirect and induced effects in other industries. This is the purpose of the Industry Contribution Analysis (ICA) functionality in IMPLAN. The basic idea is to disallow indirect and induced purchases from the industry or industries of interest in such a way that does not affect the indirect and induced effects on other industries.

For IMPLAN users, implementing ICA is as simple as selecting "Industry Contribution Analysis" as the analysis "Type" on the Impacts screen, as described in [this support article](#).   Behind the scenes, what is happening is that the rows in the A matrix for the industries of interest are being zeroed out, such that no industry or institution will be making purchases from them – while their output, GDP, and employment levels all remain unchanged, as do their purchases from all other industries.

These modifications occur as part of the analysis only; they do not affect the underlying region details data. This is a similar approach to that described in Miller and Blair's seminal input-output modeling textbook (2009, pp. 624-625 and Appendix 13.2), where the method is referred to as a "mixed endogenous-exogenous" model.

A criticism of the multi-industry contribution analysis methodology is that, if one were to perform the analysis for all industries simultaneously, the only "impacts" would be the direct impacts (i.e., the gross outputs of every industry), with no indirect or induced impacts. In other words, there would be no new information gained from such an exercise. We agree that there indeed is no point to such an exercise, since it assumes its own conclusion; but, that neither invalidates the approach nor negates its usefulness when used to examine any number of sectors less than all possible sectors.

In the technical section of this paper, we present a numerical analysis that explains IMPLAN's contribution analysis methods in reference to established input-output literature and demonstrates their consistency and appropriateness for the contribution analysis problem.

Contribution Analysis – Gross-Base Method

In gross-base contribution analysis, the goal is not to determine what would happen were a an industry to disappear from the local economy, but rather to determine the output (or employment, etc.) required by all local industries in support of that region's exogenous demands, which consist of demands by all non-internalized institutions.[2] We argue that gross-base contribution analysis is a straightforward variant of impact analysis, not a substitute for extraction-based contribution analysis. The context of the analysis determines the preferable method.

Gross-base contribution analysis, as described in Watson et al. (2015), makes one noteworthy deviation from typical impact analysis. It tabulates exogenous demands as a diagonal matrix rather than as a column vector, which gives a matrix of resulting output, rather than a column vector of gross output. It uses the same multiplier matrix as traditional impact analysis; again, the only difference is the dimensionality of the exogenous demand vector. If one models exogenous demand only for a single industry, gross-base analysis is indistinguishable from impact analysis, since both results will be column vectors of identical value.

In gross-base analysis, internalization considerations (also known in the literature as the model closure) are the same as they are for impact and extraction analysis. Internalizing affects assumptions about re-spending in the local economy: at the county level, for example, internalizing federal government implies that federal spending in a particular county is a linear function of federal tax revenue collected due to economic activity in the county.

Additionally, note that the gross-base method is not a mixed exogenous-endogenous model. For any given institution, it treats either all or none of that institution's demand as exogenous, rather than extracting certain industries, e.g., agriculture, by characterizing agriculture as exogenous and the remaining industries as endogenous. The methods recommended by Miller and Blair for "extracting" a sector involve treating some industries as exogenous and the rest as endogenous and recalculating the multiplier matrix (the Leontief inverse) after adjusting for these specifications.

The practice of multiplying exogenous demand, when defined for all institutions of a particular type, regardless of whether that demand is all final demand, only some of final demand, or only exports, by its corresponding multiplier matrix better resembles impact analysis than it resembles extraction contribution analysis.

Using either a diagonal matrix or a column vector of exogenous demands performs the same mathematical operation. The only difference is that using the diagonal matrix effectively pauses the calculation before summing each row in the result set. The sum of the elements in a row of the product of a multiplier matrix and consistently defined diagonal matrix of final demands yields the same values that would result were a column vector of exogenous demand to be used; moreover, the column vector approach gives each element that the diagonal exogenous approach yields as one of the steps in its calculation. Simply put, if one used the column vector approach and stopped before finishing it, the values would be the same as what the diagonal matrix approach gives. he individual elements of the sum, however, are informative and, depending on the context, may be worth reporting and analyzing. Watson et al. characterize the sum of the columns, rather than of the rows, as "base" output.

Accordingly, we believe that the methodology outlined in Watson et al. (2015) would be better termed base-gross contribution analysis and that it should not be confused with extraction contribution analysis, since it is incapable of consistently estimating the effect of extracting an entire sector.[3] Using the multiplier matrix recommended in Watson et al., and applying an industry's total output as the direct effect would result in an output effect larger than the total output. Similarly, using only final demand for a single industry as the direct effect would not reproduce all that industry's gross output, some of which goes to support intermediate production in other industries. In other words, it is ill-suited to answering the question of what would happen if an industry were to disappear in an input-output framework. It appears well-suited, however, to performing "base analysis" for a given specification of a model's closure (i.e., its specification as to which institutions are internalized/endogenous). Classifying exports as the only exogenous demand is the approach often adopted in gross-base analysis and has sometimes been termed "export base analysis." The method of using a diagonal vector of exogenous demands, however, can be applied to any model closure (internalization) specification. To recover total industry output for all industries, however, the exogenous demand vector needs to include all institutions not internalized in the model.

Overall, studying the components of the product of the multiplier matrix and diagonalized matrix of exports (or other combination of demand elements that have been classified as exogenous) can be informative, as explained by Watson et al., but ought to be classified as a distinct method. In short, base output for a given sector represents the effect of modeled exogenous demand for the given sector on all other sectors; gross output for a given sector, in turn, represents the effect of modeled exogenous demand for all other sectors on the given sector.

Technical Section: Internal Consistency of IMPLAN's Contribution Analysis Methods

In this section, we demonstrate via numerical examples that IMPLAN's two methods for extraction contribution analysis – single industry and multi-industry – are internally consistent with the model, consistent with the existing literature, and consistent with each other (that is, the multi-industry approach, when used for a single industry, will replicate the results of the single-industry approach).

To establish a common terminology and notation, we briefly review the basic input-output (I-O) model. The common specification of a basic I-O model holds that intermediate inputs plus final demands equals gross output. Algebraically, in matrix form, let A be a be a matrix of intermediate input ratios, I be the identity matrix, y be a column vector of final demand by sector, and x be a column vector of gross output by sector. The y vector also can be described more generally as "exogenous demand." In the basic I-O model specification, all of final demand – that is, household demand, government demand, investment demand, net inventory additions, and export demand – is classified as exogenous. What the modeler chooses to classify as exogenous is implied by what is internalized in the model. If only industries are internalized, all of final demand is exogenous (this is the specification for IMPLAN's Type 1 multipliers). If industries and households are internalized (the default specification for IMPLAN's Type SAM multipliers), then only demand by governments, capital investment, inventory, and exports are exogenous. Another way to think of this is that payments to institutions that are considered exogenous are "leaked" from the economy, whereas some portion of payments to endogenous institutions (usually industries and households in IMPLAN) are re-spent locally. According to the Leontief equation, $x = Ax + y$.

Rearranging, $x - Ax = y$, and, factoring out x, $(I - A)\,x = y$. The matrix of multipliers is found by identifying the matrix by which exogenous demand can be multiplied to reach total output, which is the inverse of $(I - A)$. Put differently, for a given amount of exogenous demand, multipliers tell how much total output needs to be produced by each industry. One multiplies the inverse of $(I - A)$ by both sides of the equation to reach the solved equation, which is $(I - A)^{-1}y = x$. This definition provides a test of the consistency of an input-output model. If any two of the three elements (multipliers, exogenous demands, and gross outputs) can be used to find the previously known third element, then the model should be considered internally consistent.

For the numerical illustrations below, we use 2014 IMPLAN data for the United States, aggregated to 5 sectors and a single household type. We begin with a basic I-O model, not yet attempting to perform a contribution analysis. Figure 1 shows the IxI SAM, with exogenous demands (final demands, in this case) highlighted in green, endogenous spending highlighted in yellow, leakages in orange, and transfers in light yellow. Exogenous demands are summed by row in the rightmost column. Figure 2 shows the corresponding multiplier matrix, which is based on an A matrix from just the endogenous industries and institutions (only the industries in this case), also shows sums of exogenous demand, and multiplies the exogenous demands by the multiplier matrix. The resulting vector is gross output, which matches the initial gross output estimate. Accordingly, the model is internally consistent when set up as a basic I-O model in which all final demand is exogenous and the multipliers are calculated according to the “Type 1” specification.

Figures 3 and 4 provide a corresponding setup based on the default IMPLAN Type SAM specification. Figure 3 shows which components of the IxI are exogenous demands – now households are no longer exogenous – and which are endogenous. The default Type SAM specification (or “model closure”) assumes that labor income payments are re-spent in the economy, less leakages to in-commuting and taxes. Figure 4 shows that the Type SAM specification is internally consistent when the corresponding multiplier matrix is estimated and applied to the new set of exogenous demands.[4]

Next, we extend the Type SAM specification to demonstrate two different methods of contribution analysis. As noted above, contribution analysis asks and answers the question of what would be the effects on all sectors’ output if an entire other sector (or sectors) were to disappear.

Miller and Blair describe this as "extracting" a sector and mathematically portray that by reclassifying the contribution sector(s)' gross output as exogenous. A necessary, but not sufficient, condition for a contribution analysis method to be consistent is that the method's result must show that the entire gross output of the contribution sector(s) is lost. That is, the result must reproduce the contribution sector(s)'s gross output. Usually, another matter of primary importance in contribution analysis is the effect on all of the other sectors. It follows that the other condition for a contribution analysis to be consistent is that the appropriate levels of multiplier effects on the non-contribution sectors are consistently estimated. As noted above, an I-O model can be deemed consistent if a set of exogenous demands and multiplier matrix reproduce gross outputs.

In the first example of contribution analysis, shown in Figures 5 and 6, the entire total output of sector 1 is exogenous, and the remaining sectors have their previous (i.e., default Type SAM-based) levels of exogenous demand. Miller and Blair explain that a viable method for estimating any number of sectors' contribution to the economy is to "zero-out" certain rows of the A matrix before calculating the multiplier matrix (p. 624-625, with general proof on p. 662). IMPLAN's method of zeroing-out RPCs achieves the same effect, as zero multiplied by the gross absorption rate for all industries equals zero, leaving zeroes in the A matrix for the contribution industry's row.5 Figure 5 identifies exogenous and endogenous demands, and Figure 6 demonstrates that the new exogenous specification (all output of sector 1, as well as previously exogenous demands for the other sectors) reproduces gross output. One can see that sector 1's gross output is used as its exogenous (or "direct impact") value, and is the same as the resulting value.

Note that Figures 5 and 6 merely show that this contribution analysis method is consistent. Figure 7 then shows the results for one sector without including exogenous demands for the non-contribution sectors, as one usually is interested in just that one sector's contribution to all other sectors' output. Figure 8 shows that the single-industry method, when used only on one industry, produces results identical to the multi-industry method as in Figure 7. Recall that the single-industry method involves either dividing the leftmost column of the multiplier matrix by the own-sector multiplier (as in Miller and Blair, p. 625, with extensions following) or the algebraically equivalent function of dividing the top row of the exogenous demand vector by that same own-sector multiplier (as recommended by IMPLAN).

Since Figures 5 and 6 demonstrate the consistency of the multi-industry contribution analysis method, and Figures 7 and 8 confirm that the multi-industry method is consistent with the single-industry method when estimating the contribution of just one industry, it remains to be shown only that the multi-industry method is indeed consistent for more than one industry. Figure 9 shows the identification of exogenous and endogenous payments. Figure 10 demonstrates consistency: by specifying the gross output of the first two sectors as exogenous and using the ordinary exogenous demand for the remaining sectors, it reproduces gross output when multiplied by a multiplier matrix based on an A matrix with the first two rows zeroed-out. Again, we rely on Miller and Blair for the formal mathematical demonstration that the method continues to be consistent for any number of “extracted”, or “contribution”, sectors.

Figures

Figure 1. SAM with Industries Internalized

IxI SAM	1	20	52	65	395	5001	6001	7001	8001	10000	11001	11002	11003	12001	12002	12003	13001	14001	14002	25001	Row Totals (Output)	Exogenous Demand
1	67,613	674	2,567	261,869	9,727	-	-	-	-	57,949	25	36	11	2,837	539	3	-	64	10,526	65,019	479,461	137,011
20	3,233	318,380	18,854	498,036	177,753	-	-	-	-	182,935	1,655	1,623	231	5,254	21,356	108	-	82,798	5,754	52,616	1,370,586	354,331
52	2,081	22,816	807	38,063	242,498	-	-	-	-	-	12,942	8,576	17,500	74,634	273	256,800	-	1,047,000	-	482	1,724,473	1,418,207
65	57,702	62,008	372,145	2,131,504	753,282	-	-	-	-	1,552,113	16,170	53,299	51,836	132,554	21,071	25,134	-	574,092	55,269	1,256,120	7,114,298	3,737,657
395	39,487	154,552	466,778	1,323,296	5,672,780	-	-	-	-	9,098,185	375,650	494,523	169,256	863,733	761,047	39,211	-	959,688	9,519	723,088	21,150,791	13,493,899
5001	51,411	179,448	364,138	971,360	7,692,005	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9,258,362	
6001	98,560	80,391	172,969	50,057	944,723	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,346,700	
7001	125,622	344,343	167,240	850,121	4,099,715	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5,587,041	
8001	548	102,160	12,104	86,082	954,905	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,155,800	
10000	7	23	85	234	1,038	8,142,969	1,287,063	1,185,100	-	255,866	2,200,169	91	31	730,058	139	7	815,500	1,038,945	2	132	15,657,459	
11001	1,164	88	119	4,973	342	1,077,700	59,637	36,400	140,097	1,420,716	4	11	10	74	17	5	458,600	794,694	191	1,360	3,996,200	
11002	0	0	0	0	1	-	-	-	-	2	603,300	0	0	0	0	0	-	0	0	0	603,304	
11003	0	0	1	2	7	-	-	-	-	11	264,600	1	0	1	1	0	-	1	0	1	264,625	
12001	1,011	2,965	9,473	27,411	115,085	18,900	-	40,100	1,015,703	640,867	502,424	10,038	3,436	17,539	15,438	796	62,100	497,636	223	14,855	2,996,000	
12002	-	-	-	-	-	-	-	-	-	-	-	-	-	763,467	-	-	-	96,300	-	-	859,767	
12003	-	-	-	-	-	-	-	-	-	-	-	-	-	331,300	-	-	-	0	-	-	331,300	
13001	-	-	-	-	-	-	-	1,800,369	-	-	-	-	-	-	-	-	-	-	-	-	1,800,369	
14001	194	609	2,298	6,300	27,944	-	-	2,534,700	-	1,624,571	1,855	2,443	836	4,265	3,750	194	464,169	4,703	44	722,414	5,401,289	
14002	1,055	426	955	9,129	2,115	-	-	-	-	4,714	41	132	126	366	77	61	-	78,566	281	3,973	102,018	
25001	29,770	101,703	133,941	855,862	456,872	18,793	0	(9,628)	-	819,531	17,363	32,533	21,351	69,917	36,058	8,982	-	226,802	20,208	-	2,840,059	

Figure 2. Basic I-O Model (all final demand exogenous, not doing contribution analysis for any sector)

Multiplier Matrix (Type 1 in Implan)									
(I-A)^(-1)						y	(I-A)^(-1)*y	x	
Inverse	1	20	52	65	395	Exogenous Demand	Inverse * Exogenous Demand	Initial Gross Output	Difference
1	1.174	0.005	0.017	0.063	0.004	137,011	479,461	479,461	-
20	0.032	1.315	0.051	0.140	0.023	354,331	1,370,586	1,370,586	-
52	0.009	0.025	1.009	0.015	0.017	1,418,207	1,724,473	1,724,473	-
65	0.216	0.106	0.341	1.473	0.078	3,737,657	7,114,298	7,114,298	-
395	0.195	0.240	0.469	0.409	1.397	13,493,899	21,150,791	21,150,791	-

Figure 3. SAM with Industries and Households Internalized

IxI SAM	1	20	52	65	395	5001	6001	7001	8001	10000	11001	11002	11003	12001	12002	12003	13001	14001	14002	25001	Row Totals (Output)	Exogenous Demand
1	67,613	674	2,567	261,869	9,727	-	-	-	-	57,949	25	36	11	2,837	539	3	-	64	10,526	65,019	479,461	79,061
20	3,233	318,380	18,854	498,036	177,753	-	-	-	-	182,935	1,655	1,623	231	5,254	21,356	108	-	82,798	5,754	52,616	1,370,586	171,395
52	2,081	22,816	807	38,063	242,498	-	-	-	-	-	12,942	8,576	17,500	74,634	273	256,800	-	1,047,000	-	482	1,724,473	1,418,207
65	57,702	62,008	372,145	2,131,504	753,282	-	-	-	-	1,552,113	16,170	53,299	51,836	132,554	21,071	25,134	-	574,092	55,269	1,256,120	7,114,298	2,185,544
395	39,487	154,552	466,778	1,323,296	5,672,780	-	-	-	-	9,098,185	375,650	494,523	169,256	863,733	761,047	39,211	-	959,688	9,519	723,088	21,150,791	4,395,714
5001	51,411	179,448	364,138	971,360	7,692,005	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9,258,362	-
6001	98,560	80,391	172,969	50,057	944,723	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,346,700	-
7001	125,622	344,343	167,240	850,121	4,099,715	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5,587,041	
8001	548	102,160	12,104	86,082	954,905	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,155,800	
10000	7	23	85	234	1,038	8,142,969	1,287,063	1,185,100	-	255,866	2,200,169	91	31	730,058	139	7	815,500	1,038,945	2	132	15,657,459	5,970,174
11001	1,164	88	119	4,973	342	1,077,700	59,637	36,400	140,097	1,420,716	4	11	10	74	17	5	458,600	794,694	191	1,360	3,996,200	
11002	0	0	0	0	1	-	-	-	-	2	603,300	0	0	0	0	0	-	0	0	0	603,304	
11003	0	0	1	2	7	-	-	-	-	11	264,600	1	0	1	1	0	-	1	0	1	264,625	
12001	1,011	2,965	9,473	27,411	115,085	18,900	-	40,100	1,015,703	640,867	502,424	10,038	3,436	17,539	15,438	796	62,100	497,636	223	14,855	2,996,000	
12002	-	-	-	-	-	-	-	-	-	-	-	-	-	763,467	-	-	-	96,300	-	-	859,767	
12003	-	-	-	-	-	-	-	-	-	-	-	-	-	331,300	-	-	-	0	-	-	331,300	
13001	-	-	-	-	-	-	-	1,800,369	-	-	-	-	-	-	-	-	-	-	-	-	1,800,369	
14001	194	609	2,298	6,300	27,944	-	-	2,534,700	-	1,624,571	1,855	2,443	836	4,265	3,750	194	464,169	4,703	44	722,414	5,401,289	
14002	1,055	426	955	9,129	2,115	-	-	-	-	4,714	41	132	126	366	77	61	-	78,566	281	3,973	102,018	
25001	29,770	101,708	133,941	855,862	456,872	18,793	0	(9,628)	-	819,531	17,363	32,533	21,351	69,917	36,058	8,982	-	226,802	20,208	-	2,840,059	

Figure 4. Default IMPLAN Type SAM Setup (Labor Income and Households Internalized)

Multiplier Matrix (Type SAM in Implan)												
(I-A)^(-1)									y	(I-A)^(-1)*y	x	
Inverse	1	20	52	65	395	5001	6001	10000	Exogenous Demand	Inverse * Exogenous Demand	Initial Gross Output	Difference
1	1.183	0.012	0.027	0.071	0.015	0.018	0.020	0.021	79,061	479,461	479,461	-
20	0.062	1.338	0.085	0.165	0.058	0.059	0.064	0.067	171,395	1,370,586	1,370,586	-
52	0.017	0.031	1.018	0.022	0.026	0.016	0.017	0.018	1,418,207	1,724,473	1,724,473	-
65	0.353	0.208	0.497	1.590	0.239	0.268	0.292	0.305	2,185,544	7,114,298	7,114,298	-
395	0.799	0.690	1.158	0.923	2.106	1.186	1.289	1.348	4,395,714	21,150,791	21,150,791	-
5001	0.477	0.462	0.718	0.587	0.813	1.481	0.523	0.547	-	9,258,362	9,258,362	-
6001	0.287	0.116	0.168	0.079	0.105	0.064	1.069	0.072	-	1,346,700	1,346,700	-
10000	0.706	0.527	0.805	0.602	0.829	1.386	1.506	1.576	5,970,174	15,657,459	15,657,459	-

Figure 5. SAM with Industries (except Industry 1) and Households Internalized

IxI SAM	1	20	52	65	395	5001	6001	7001	8001	10000	11001	11002	11003	12001	12002	12003	13001	14001	14002	25001	Row Totals (Output)	Exogenous Demand
1	67,613	674	2,567	261,869	9,727	-	-	-	-	57,949	25	36	11	2,837	539	3	-	64	10,526	65,019	479,461	479,461
20	3,233	318,380	18,854	498,036	177,753	-	-	-	-	182,935	1,655	1,623	231	5,254	21,356	108	-	82,798	5,754	52,616	1,370,586	171,395
52	2,081	22,816	807	38,063	242,498	-	-	-	-	-	12,942	8,576	17,500	74,634	273	256,800	-	1,047,000	-	482	1,724,473	1,418,207
65	57,702	62,008	372,145	2,131,504	753,282	-	-	-	-	1,552,113	16,170	53,299	51,836	132,554	21,071	25,134	-	574,092	55,269	1,256,120	7,114,298	2,185,544
395	39,487	154,552	466,778	1,323,296	5,672,780	-	-	-	-	9,098,185	375,650	494,523	169,256	863,733	761,047	39,211	-	959,688	9,519	723,088	21,150,791	4,395,714
5001	51,411	179,448	364,138	971,360	7,692,005	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9,258,362	-
6001	98,560	80,391	172,969	50,057	944,723	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,346,700	-
7001	125,622	344,348	167,240	850,121	4,099,715	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5,587,041	
8001	548	102,160	12,104	86,082	954,905	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,155,800	
10000	7	23	85	234	1,038	8,142,969	1,287,063	1,185,100	-	255,866	2,200,169	91	31	730,058	139	7	815,500	1,038,945	2	132	15,657,459	5,970,174
11001	1,164	88	119	4,973	342	1,077,700	59,637	36,400	140,097	1,420,716	4	11	10	74	17	5	458,600	794,694	191	1,360	3,996,200	
11002	0	0	0	0	1	-	-	-	-	2	603,300	0	0	0	0	0	-	0	0	0	603,304	
11003	0	0	1	2	7	-	-	-	-	11	264,600	1	0	1	1	0	-	1	0	1	264,625	
12001	1,011	2,965	9,473	27,411	115,085	18,900	-	40,100	1,015,703	640,867	502,424	10,088	3,436	17,539	15,438	796	62,100	497,636	223	14,855	2,996,000	
12002	-	-	-	-	-	-	-	-	-	-	-	-	-	763,467	-	-	-	96,300	-	-	859,767	
12003	-	-	-	-	-	-	-	-	-	-	-	-	-	331,300	-	-	-	0	-	-	331,300	
13001	-	-	-	-	-	-	-	1,800,369	-	-	-	-	-	-	-	-	-	-	-	-	1,800,369	
14001	194	609	2,298	6,300	27,944	-	-	2,534,700	-	1,624,571	1,855	2,443	836	4,265	3,750	194	464,169	4,703	44	722,414	5,401,289	
14002	1,055	426	955	9,129	2,115	-	-	-	-	4,714	41	132	126	366	77	61	-	78,566	281	3,973	102,018	
25001	29,770	101,708	133,941	855,862	456,872	18,793	0	(9,628)	-	819,531	17,363	32,583	21,351	69,917	36,058	8,982	-	226,802	20,208	-	2,840,059	

Figure 6. Contribution of Sector 1 – showing that all exogenous demands reproduce all outputs – using default Type SAM model and multi-industry method (adjust byproducts and zero-out row of A matrix)

Multiplier Matrix (Type SAM in Implan)													
(I-A)^(-1)									y		(I-A)^(-1)*y	x	
Inverse	1	20	52	65	395	5001	6001	10000	Exogenous Demand		Inverse * Exogenous Demand	Initial Gross Output	Difference
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	479,461	*Gross output	479,461	479,461	-
20	0.053	1.337	0.084	0.161	0.057	0.058	0.063	0.066	171,395	*exogenous demand	1,370,586	1,370,586	-
52	0.015	0.031	1.018	0.021	0.026	0.016	0.017	0.018	1,418,207	*exogenous demand	1,724,473	1,724,473	-
65	0.298	0.204	0.489	1.568	0.234	0.263	0.286	0.299	2,185,544	*exogenous demand	7,114,298	7,114,298	-
395	0.676	0.682	1.140	0.875	2.096	1.174	1.275	1.334	4,395,714	*exogenous demand	21,150,791	21,150,791	-
5001	0.404	0.457	0.707	0.558	0.807	1.474	0.515	0.539	-	*exogenous demand	9,258,362	9,258,362	-
6001	0.242	0.113	0.161	0.062	0.101	0.059	1.064	0.067	-	*exogenous demand	1,346,700	1,346,700	-
10000	0.596	0.519	0.789	0.559	0.820	1.375	1.494	1.564	5,970,174	*exogenous demand	15,657,459	15,657,459	-

Figure 7. Contribution of Sector 1 – without including exogenous demands for other sectors – using default Type SAM model and multi-industry method (adjust byproducts and zero-out row of A matrix)

Multiplier Matrix (Type SAM in Implan)											
(I-A)^(-1)									y		(I-A)^(-1)*y
Inverse	**1**	**20**	**52**	**65**	**395**	**5001**	**6001**	**10000**	**Exogenous Demand**		**Inverse * Exogenous Demand**
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	479,461	*Gross output	479,461
20	0.053	1.337	0.084	0.161	0.057	0.058	0.063	0.066	-		25,259
52	0.015	0.031	1.018	0.021	0.026	0.016	0.017	0.018	-		6,984
65	0.298	0.204	0.489	1.568	0.234	0.263	0.286	0.299	-		143,115
395	0.676	0.682	1.140	0.875	2.096	1.174	1.275	1.334	-		323,895
5001	0.404	0.457	0.707	0.558	0.807	1.474	0.515	0.539	-		193,526
6001	0.242	0.113	0.161	0.062	0.101	0.059	1.064	0.067	-		116,217
10000	0.596	0.519	0.789	0.559	0.820	1.375	1.494	1.564	-		285,983

Figure 8. Contribution of Sector 1, using default Type SAM model and single-industry method (adjust level of exogenous demand)

Multiplier Matrix (Type SAM in Implan)											
(I-A)^(-1)									Contribution Sector	y	(I-A)^(-1)*y
Inverse	**1**	**20**	**52**	**65**	**395**	**5001**	**6001**	**10000**	**Gross Output / Detail Multiplier**	**Exogenous Demand**	**Inverse * Exogenous Demand**
1	1.183	0.012	0.027	0.071	0.015	0.018	0.020	0.021	405,349	405,349	479,461
20	0.062	1.338	0.085	0.165	0.058	0.059	0.064	0.067		-	25,259
52	0.017	0.031	1.018	0.022	0.026	0.016	0.017	0.018		-	6,984
65	0.353	0.208	0.497	1.590	0.239	0.268	0.292	0.305		-	143,115
395	0.799	0.690	1.158	0.923	2.106	1.186	1.289	1.348		-	323,895
5001	0.477	0.462	0.718	0.587	0.813	1.481	0.523	0.547		-	193,526
6001	0.287	0.116	0.168	0.079	0.105	0.064	1.069	0.072		-	116,217
10000	0.706	0.527	0.805	0.602	0.829	1.386	1.506	1.576		-	285,983

Figure 9. SAM with Industries (except Industries 1 and 2) and Households Internalized

Ind SAM	1	20	52	65	395	5001	6001	7001	8001	10000
1	67,613	674	2,567	261,869	9,727	-	-	-	-	57,949
20	3,233	318,380	18,854	498,036	177,753	-	-	-	-	182,935
52	2,081	22,816	807	38,063	242,498	-	-	-	-	-
65	57,702	62,008	372,145	2,131,504	753,282	-	-	-	-	1,552,113
395	39,487	154,552	466,778	1,323,296	5,672,780	-	-	-	-	9,098,185
5001	51,411	179,448	364,138	971,360	7,692,005	-	-	-	-	-
6001	98,560	80,391	172,969	50,057	944,723	-	-	-	-	-
7001	125,622	344,343	167,240	850,121	4,099,715	-	-	-	-	-
8001	548	102,160	12,104	86,082	954,905	-	-	-	-	-
10000	7	23	85	234	1,038	8,142,969	1,287,063	1,185,100	-	255,866
11001	1,164	88	119	4,973	342	1,077,700	59,637	36,400	140,097	1,420,716
11002	0	0	0	0	1	-	-	-	-	2
11003	0	0	1	2	7	-	-	-	-	11
12001	1,011	2,965	9,473	27,411	115,085	18,900	-	40,100	1,015,703	640,867
12002	-	-	-	-	-	-	-	-	-	-
12003	-	-	-	-	-	-	-	-	-	-
13001	-	-	-	-	-	-	-	1,800,369	-	-
14001	194	609	2,298	6,300	27,944	-	-	2,534,700	-	1,624,571
14002	1,055	426	955	9,129	2,115	-	-	-	-	4,714
25001	29,770	101,708	133,941	855,862	456,872	18,793	0	(9,628)	-	819,531

Ind SAM	11001	11002	11003	12001	12002	12003	13001	14001	14002	25001	Row Totals (Output)	Exogenous Demand
1	25	36	11	2,837	539	3	-	64	10,526	65,019	479,461	479,461
20	1,655	1,623	231	5,234	21,356	108	-	82,798	5,754	52,616	1,370,586	1,370,586
52	12,942	8,576	17,500	74,634	273	256,800	-	1,047,000	-	482	1,724,473	1,418,207
65	16,170	53,299	51,836	132,554	21,071	25,134	-	574,092	55,269	1,256,120	7,114,298	2,185,544
395	375,650	494,523	169,256	863,733	761,047	39,211	-	959,688	9,519	723,088	21,150,791	4,395,714
5001	-	-	-	-	-	-	-	-	-	-	9,258,362	-
6001	-	-	-	-	-	-	-	-	-	-	1,346,700	-
7001	-	-	-	-	-	-	-	-	-	-	5,587,041	
8001	-	-	-	-	-	-	-	-	-	-	1,155,800	
10000	2,200,169	91	31	730,058	139	7	815,500	1,038,945	2	132	15,657,459	5,970,174
11001	4	11	10	74	17	5	458,600	794,694	191	1,360	3,996,200	
11002	603,300	0	0	0	0	0	-	0	0	0	603,304	
11003	264,600	1	0	1	1	0	-	1	0	1	264,625	
12001	502,424	10,088	3,436	17,539	15,438	796	62,100	497,636	223	14,855	2,996,000	
12002	-	-	-	763,467	-	-	-	96,300	-	-	859,767	
12003	-	-	-	331,300	-	-	-	0	-	-	331,300	
13001	-	-	-	-	-	-	-	-	-	-	1,800,369	
14001	1,855	2,443	836	4,265	3,750	194	464,169	4,703	44	722,414	5,401,289	
14002	41	132	126	366	77	61	-	78,566	281	3,973	102,018	
25001	17,363	32,533	21,351	69,917	36,058	8,982	-	226,802	20,208	-	2,840,059	

Figure 10. Demonstration of Consistency: (I-A)-1 Table of Multipliers for SAM with Industries and Households Internalized (Type SAM multipliers), showing that all exogenous demands reproduce all outputs with two-industry contribution

Multiplier Matrix (Type SAM in Implan)													
(I-A)^(-1)									y		(I-A)^(-1)*y	x	
Inverse	1	20	52	65	395	5001	6001	10000	Exogenous Demand		Inverse * Exogenous Demand	Initial Gross Output	Difference
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	479,461	*Gross output	479,461	479,461	-
20	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1,370,586	*Gross output	1,370,586	1,370,586	-
52	0.013	0.023	1.016	0.017	0.025	0.014	0.016	0.016	1,418,207	*exogenous demand	1,724,473	1,724,473	-
65	0.290	0.153	0.476	1.544	0.226	0.254	0.276	0.289	2,185,544	*exogenous demand	7,114,298	7,114,298	-
395	0.649	0.510	1.097	0.793	2.067	1.144	1.243	1.301	4,395,714	*exogenous demand	21,150,791	21,150,791	-
5001	0.386	0.342	0.679	0.503	0.788	1.454	0.493	0.516	-	*exogenous demand	9,258,362	9,258,362	-
6001	0.238	0.085	0.154	0.048	0.096	0.054	1.059	0.062	-	*exogenous demand	1,346,700	1,346,700	-
10000	0.576	0.388	0.757	0.496	0.798	1.353	1.470	1.538	5,970,174	*exogenous demand	15,657,459	15,657,459	-

References

Miller, R.E. and P.D. Blair. 2009. Input-Output Analysis: Foundations and Extensions, Second Edition. New York: Cambridge University Press.

Watson, P., S. Cooke, D. Kay, and G. Alward. 2015. A Method for Improving Economic Contribution Studies for Regional Analysis. Journal of Regional Policy and Analysis, 45(1): 1-15

IMPLAN

IMPLAN has been redefining the field of economics for over 40 years. Created by academics to serve the needs of the United States Forest Service, it has been transformed today to serve as a solution-provider for anyone interested in understanding their economy.

16905 Northcross Drive, Suite 120
Huntersville, NC 28078
(800) 507-9426

IMPLAN